MATH 140A Review: Quantifiers and Negation

1. Negate the following statement: Let f be a real-valued function on $\mathbb{R}$. For every $x \in \mathbb{R}$, for every $\epsilon > 0$, there exists $\delta > 0$ such that if y satisfies $|x - y| < \delta$, then $|f(x) - f(y)| < \epsilon$.

   Hint:

   $$\forall x \in \mathbb{R}, \forall \epsilon \in (0, \infty), \exists \delta \in (0, \infty), \forall y \in \mathbb{R}, \text{ if } |x - y| < \delta, \text{ then } |f(x) - f(y)| < \epsilon.$$

   Solution: The negation is:

   $$\exists x \in \mathbb{R}, \exists \epsilon \in (0, \infty), \forall \delta \in (0, \infty), \exists y \in \mathbb{R} \text{ such that } |x - y| \geq \delta \text{ and } |f(x) - f(y)| \geq \epsilon.$$

   In words, the negation is: there exists $x \in \mathbb{R}$ and $\epsilon \in (0, \infty)$ such that for all $\delta \in (0, \infty)$, there exists $y \in \mathbb{R}$ such that $|x - y| \geq \delta$ and $|f(x) - f(y)| \geq \epsilon$.

2. Negate the following statement: Let f be a real-valued function on $\mathbb{R}$. For every $\epsilon > 0$, there exists $\delta > 0$ such that for all $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| < \epsilon$.

   Solution: There exists $\epsilon > 0$ such that for all $\delta > 0$, there exists real numbers x and y such that $|x - y| \geq \delta$ and $|f(x) - f(y)| \geq \epsilon$.

3. Negate the following statement: Let A be a subset of $\mathbb{R}$ and let $M \in \mathbb{R}$. For all $y \in \mathbb{R}$, if $y < M$, then there exists $x \in A$ such that $x > y$.

   Solution: There exists $y \in \mathbb{R}$ such that $y < M$ and for every $x \in A$, we have $x \leq y$.